1. Show that the solution below is an exact integral manifold for the dynamic system shown for all epsilon.

$$
\begin{aligned}
& \psi_{d e}=V_{s} \cos \left(\delta-\theta_{v s}\right) \\
& \psi_{q e}=-V_{s} \sin \left(\delta-\theta_{v s}\right) \\
& \varepsilon \frac{d \psi_{d e}}{d t}=\left(1+\frac{\varepsilon}{T_{s}} \omega_{t}\right) \psi_{q e}+V_{s} \sin \left(\delta-\theta_{v s}\right) \\
& \varepsilon \frac{d \psi_{q e}}{d t}=-\left(1+\frac{\varepsilon}{T_{s}} \omega_{t}\right) \psi_{d e}+V_{s} \cos \left(\delta-\theta_{v s}\right) \\
& T_{s} \frac{d \delta}{d t}=\omega_{t}
\end{aligned}
$$

2. Find an approximate integral manifold for $z$ as a function of $x$ in the following system keeping epsilon but neglecting order epsilon squared and higher:

$$
\begin{gathered}
\frac{d x}{d t}=-x+z \\
\varepsilon \frac{d z}{d t}=2 x^{2}-z
\end{gathered}
$$

## 3. Text problem 5.6

The two-axis machine model with data given below is to be used for problems 4. and 5. The parameters are either in seconds or per unit on the base of a 555 MVA (3-phase), 24 KV (line-line), 60 Hz , 2-pole (3600 RPM) synchronous machine:

| $\mathrm{R}_{\mathrm{S}}=0.003$ | $X_{15}=0.15$ | $X_{m d}=1.66$ | $X_{m q}=1.61$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{R}_{\mathrm{fd}}=0.0006$ | $\mathrm{X}_{\mathrm{fd}}=1.824$ | $\mathrm{C}_{\mathrm{d}}=\mathrm{C}_{\mathrm{q}}=1$ | $X_{d}{ }^{\prime}=0.30$ |  |
| $\mathrm{R}_{1 q}=0.00619$ | $\mathrm{X}_{1 q}=2.3452$ | $X_{q}{ }^{\prime}=0.65$ | Neglect saturation |  |
| $\mathrm{T}_{\text {do }}{ }^{\prime}=8.0 \mathrm{sec}$ | $\mathrm{T}_{\text {qo }}{ }^{\prime}=1.0 \mathrm{sec}$ | $\mathrm{H}=3.5 \mathrm{sec}$ | Neglect friction and windage |  |
| $\mathrm{K}_{\mathrm{A}}=200$. | $\mathrm{T}_{\mathrm{A}}=0.04 \mathrm{sec}$ | $\mathrm{K}_{\mathrm{F}}=0.03$ | $\mathrm{T}_{\mathrm{F}}=1.0 \mathrm{sec}$ | $K_{E}=1.0$ |
| $\mathrm{T}_{\mathrm{E}}=0.7 \mathrm{sec}$ | $V_{R}{ }^{\text {min }}=-5.0$ | $V_{R}{ }^{\text {max }}=5.0$ | $\mathrm{S}_{\mathrm{E}}=0.0$ |  |
| $\mathrm{T}_{\text {CH }}=0.05$ | $\mathrm{T}_{\text {sv }}=0.1 \mathrm{sec}$ | $\mathrm{R}_{\mathrm{D}}=0.05$ | $\mathrm{Psv}^{\text {min }}=0$. | $\mathrm{Psv}^{\text {max }}=1$. |

4. The machine is running at synchronous speed with an open-circuited terminal voltage equal to 1.01 per unit.
(a) Find the initial conditions of all dynamic states plus the control inputs $\mathrm{V}_{\text {ref }}$ and $\mathrm{P}_{\mathrm{c}}$.
(b) Keep the control inputs constant and find the dynamic response when it is loaded with a pure resistive load of 2.0 per unit ( Y connected) per phase. Run the simulation for 1 second with no disturbance, then add the load at $t=1.0 \mathrm{sec}$ and run the simulation for another 10 seconds. Use whatever numerical integration method and step size that appears reasonable to you. Plot the speed, angle delta, terminal voltage magnitude and angle (theta) vs time. Submit your solution code and plots.
(c) Check the steady-state solution above by solving for speed and all the dynamic states from the algebraic equations obtained by setting all derivatives to zero except delta. Compute the angle difference theta (angle on terminal voltage) minus delta.
5. Rather than a static load as in 4. the machine is now connected to an infinite bus ( $\mathrm{V}=1$ pu angle zero on machine base) through an equivalent transmission line with $R_{e}=0$ and $X_{e p}=0.5$ pu (on machine base).
(a) Find the initial steady-state values of all states and inputs if $\omega=\omega_{s}, V_{t}=1.02$ pu angle 20 degrees.
(b) Keep the control inputs constant and find the dynamic response when the line reactance is suddenly changed to $X_{\text {ep }}=0.8 \mathrm{pu}$ (simulates the outage of a line) at $t=1 \mathrm{sec}$. Run the simulation for 10 more seconds. Use whatever numerical integration method and time step that appears reasonable to you. Plot the generator current and terminal voltage magnitudes plus the generator speed. Submit your solution code and plots.
(c) Using the original $X_{\text {ep }}$, and the original steady state, play with the changed value of $X_{e p}$ to see at what value the machine will become unstable because the new modified equivalent line is too weak to transmit the amount of power initially given. Submit your solution code and plots (needed to decide).
